#### Charge Conjugation Invariance of the Vacuum and the Cosmological Constant Problem

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#### **Abstract**

We propose a method of field quantization which uses an indefinite metric in a Hilbert space of state vectors. The action for gravity and the standard model includes, as well as the positive energy fermion and boson fields, negative energy fields. The Hamiltonian for the action leads through charge conjugation invariance symmetry of the vacuum to a cancellation of the zeropoint vacuum energy and a vanishing cosmological constant in the presence of a gravitational field. To guarantee the stability of the vacuum, we introduce a Dirac sea 'hole' theory of quantization for gravity as well as the standard model. The vacuum is defined to be fully occupied by negative energy particles with a hole in the Dirac sea, corresponding to an anti-particle. We postulate that the negative energy bosons in the vacuum satisfy a para-statistics that leads to a para-Pauli exclusion principle for the negative energy bosons in the vacuum, while the positive energy bosons in the Hilbert space obey the usual Bose-Einstein statistics. This assures that the vacuum is stable for both fermions and bosons. Restrictions on the para-operator Hamiltonian density lead to selection rules that prohibit positive energy para-bosons from being observable. The problem of deriving a positive energy spectrum and a consistent unitary field theory from a pseudo-Hermitian Hamiltonian is investigated.

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### 1 Introduction

The cosmological constant problem is considered to be one of the major problems of modern physics [1]. The particle physics origin of the problem arises because of the quartic divergence of the zero-point vacuum energy in the presence of a gravitational field. The constant zero-point energy cannot be shifted to zero in the action due to

the universal coupling of gravity to energy including vacuum energy. It is hoped that a natural symmetry exists that explains why the cosmological constant is zero or small. However, the obvious candidates, supersymmetry and conformal invariance, cannot supply this solution, for they are badly broken in Nature. It is possible that the standard second quantization method, that forms the basis for modern quantum field theory, possesses a fundamental flaw in its physical interpretation of the vacuum. Due to this possible faulty understanding of the vacuum state in quantum field theory, we are led to two important difficulties inherent in the theory:

1) a generic infinity of the zero-point vacuum energy in the presence of a gravitational field and, 2) the lack of renormalizability of perturbative quantum gravity.

Linde [2, 3] suggested some time ago that that a novel symmetry could be introduced between two different universes with independent spacetimes. The symmetry transforms the action S into -S, so that the positive energy of one spacetime is transformed into the negative energy of the other spacetime. The action S is symmetric under the transformation of the matter field  $\phi(x) \to \overline{\phi}(x)$  and the metric  $g_{\mu\nu}(x) \to \overline{g}_{\mu\nu}(x)$ . A consequence of this symmetry is the invariance under the change of the values of the effective potentials  $V(\phi) \to V(\phi) + c$ ,  $V(\overline{\phi}) \to V(\overline{\phi}) + c$ , where c is a constant. However, in order to avoid negative probabilities and the instability of the vacuum associated with negative energy particles, Linde postulated that the positive and negative energy fields do not interact. This postulate introduces an extreme and unnatural fine-tuning of the model.

Recently, Kaplan and Sundrum [4] have introduced a symmetry which transforms positive energy into negative energy through a projection operator P. As in the case of Linde's two - universe proposal, the authors postulate that to avoid a breakdown of the vacuum due to the negative energy particles, the two copies of the standard model matter fields, corresponding to positive and negative energy particles, interact only weakly through gravity. To prevent excessively rapid decay of the vacuum, it is also postulated that gravitational Lorentz invariance breaks down at short distances.

In the following, we shall pursue a different approach to the solution of the cosmological constant problem, and the possible avoidance of the lack of renormalizability of perturbative quantum gravity, by following a method of quantizing fields introduced by Dirac in 1942 [5] and investigated in detail by Pauli [6] and Sudarshan and collaborators [7, 8, 9]. We investigate a method of field quantization motivated by Bender and collaborators [10, 11, 12, 13, 14, 15, 16, 17] that can implement a positive energy spectrum for a pseudo-Hermitian Hamiltonian, involving an indefinite metric in Hilbert space, and a unitary S-matrix. A para-statistics is invoked for negative energy bosons and their corresponding "holes" in the vacuum, which together with the negative energy Pauli exclusion principle for fermions and a para-Pauli exclusion principle for negative energy bosons, leads to a stability of the vacuum.

The charge conjugation invariance of the vacuum state leads to a cancellation of the zero-point vacuum energy in the presence of a gravitational field and to the vanishing of the cosmological constant.

## 2 Indefinite Metric in Hilbert Space

Dirac [5, 6] generalized field quantization by introducing an indefinite metric in the Hilbert space of the state vectors. The normalization of a state vector  $\Psi$  is normally defined by

$$\mathcal{N}_{+} = \int dq \Psi^* \Psi, \tag{1}$$

where  $\Psi^*$  is the complex conjugate of  $\Psi$ . The scalar product of two complex state vectors  $\Phi$  and  $\Psi$  is given by

$$\mathcal{B}_{+} = \int dq \Phi^* \Psi. \tag{2}$$

Instead, we consider the more general bilinear form

$$\mathcal{B} = \int dq \Phi^* \eta \Psi, \tag{3}$$

in which the operator  $\eta$  is an Hermitian operator to guarantee real normalization values.

The expectation value of an observable  $\mathcal{O}$  described by a linear operator is now defined by

$$\langle \mathcal{O} \rangle = \int dq \Psi^* \eta \mathcal{O} \Psi. \tag{4}$$

The generalization of the standard Hermitian conjugate operator  $\mathcal{O}=\mathcal{O}^\dagger$  is given by the adjoint operator

$$\tilde{\mathcal{O}} = \eta^{-1} \mathcal{O}^{\dagger} \eta^{\dagger} = \eta^{-1} \mathcal{O}^{\dagger} \eta. \tag{5}$$

All physical observables have to be self-adjoint,  $\tilde{\mathcal{O}} = \mathcal{O}$ , to guarantee that their expectation values are real. In particular, the Hamiltonian operator H has to be self-adjoint,  $\tilde{H} = H$ , which has the consequence that

$$\frac{d}{dt} \int dq \Psi^* \eta \Psi = i \Psi^* \eta (\tilde{H} - H) \Psi = 0, \tag{6}$$

guaranteeing the conservation of the normalization with time.

A linear transformation in the Hilbert space

$$\Psi' = \mathcal{S}\Psi,\tag{7}$$

requires that

$$\eta' = \mathcal{S}^{\dagger} \eta \mathcal{S}. \tag{8}$$

This assures that the normalization of the state vector is invariant

$$\int dq \Psi^{*'} \eta' \Psi' = \int dq \Psi^* \eta \Psi. \tag{9}$$

Then we have

$$\mathcal{O}' = \mathcal{S}^{-1}\mathcal{O}\mathcal{S}, \quad \tilde{\mathcal{O}}' = \eta'^{-1}\mathcal{O}^{\dagger'}\eta' = \mathcal{S}^{-1}\tilde{\mathcal{O}}\mathcal{S}, \tag{10}$$

and the expectation value of the operator  $\mathcal{O}$  is invariant

$$\int dq \Psi^{*'} \eta' \mathcal{O}' \Psi' = \int dq \Psi^* \eta \mathcal{O} \Psi. \tag{11}$$

We note that the usual Hermitian property of an operator is not invariant with respect to the S transformation, whereas the quality of being self-adjoint is invariant.

A transformation of the Hermitian matrix  $\eta$  to normal, diagonal form can have the values 1 or -1. The positive definite form (1) yields a unit matrix and positive probabilities. However, in general positive eigenvalues can have negative probabilities, i.e. one can introduce negative probabilities that certain positive eigenvalues of an observable are realized. We shall discuss the possibility of obtaining a physical interpretation of quantum field theory in which the S-matrix is unitary and only a positive energy spectrum is observed when the pseudo-Hermitian Hamiltonian is quantized.

### 3 The Action and Field Quantization

The action takes the form  $(c = \hbar = 1)$ :

$$S = S_{\text{Grav}} + S_M(\phi) + S_M(\chi), \tag{12}$$

where

$$S_{\text{Grav}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [(R - 2\Lambda_0) - (\overline{R} - 2\overline{\Lambda}_0)]. \tag{13}$$

The R denotes the normal Ricci scalar associated with positive energy gravitons, while  $\overline{R}$  is associated with negative energy gravitons. Moreover,  $\Lambda_0$  and  $\overline{\Lambda}_0$  denote the "bare" cosmological constants corresponding to positive and negative energy gravitons. The  $\phi$  and  $\chi$  fields denote positive and negative energy matter fields, respectively.

We define an effective cosmological constant

$$\Lambda_{\text{eff}} = \Lambda_{0\text{eff}} + \Lambda_{\text{vac}},\tag{14}$$

where  $\Lambda_{0\text{eff}} = \Lambda_0 - \overline{\Lambda}_0$ ,  $\Lambda_{\text{vac}} = 8\pi G \rho_{\text{vac}}$  and  $\rho_{\text{vac}}$  denotes the vacuum density. We expand the metric tensor  $g_{\mu\nu}$  about Minkowski flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2), \tag{15}$$

where  $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ . We begin with the lowest weak field approximation for which  $\sqrt{-g} = 1$ .

Let us consider as a first simple case a real scalar field  $\phi(x)$  in the absence of interactions. The action is

$$S_{\phi} = \frac{1}{2} \int d^4x \left[ \partial_{\mu} \phi \partial^{\mu} \phi - \mu^2 \phi^2 \right], \tag{16}$$

and  $\phi$  satisfies the wave equation

$$(\partial^{\mu}\partial_{\mu} + \mu^2)\phi = 0. \tag{17}$$

As is well known, this equation has both positive and negative energy solutions, as is the case with the Dirac equation [20].

The Hamiltonian for positive energy scalar particles is given by

$$H = \frac{1}{2} \int d^3x \left[ (\vec{\nabla}\phi)^2 + (\partial_0\phi)^2 + \mu^2\phi^2 \right]. \tag{18}$$

The usual decomposition of the field  $\phi$  into Fourier components is given by

$$\phi(x) = V^{-1/2} \sum_{k} (2k_0)^{-1/2} \{ \phi(k) \exp[i(\vec{k} \cdot \vec{x} - k_0 x_0)] + \phi^*(k) \exp[i(-\vec{k} \cdot \vec{x} + k_0 x_0)] \}, \quad (19)$$

where  $k_0$  is positive,  $k_0 = +(\vec{k}^2 + \mu^2)^{1/2}$ . The standard quantization is based on the commutator

$$[\phi(k), \phi^*(k)] = 1,$$
 (20)

and gives for the energy

$$E = \sum_{k} k_0 \left[ \frac{1}{2} + N_k \right]. \tag{21}$$

For the vacuum (ground state),  $N_k = 0$  and we obtain the zero-point vacuum energy

$$E_0 = \frac{1}{2} \sum_{k} k_0 = \frac{1}{2(2\pi)^3} \int d^3k \sqrt{\vec{k}^2 + \mu^2}.$$
 (22)

The zero-point vacuum energy  $E_0$  diverges quartically and is the root of the cosmological constant problem in the presence of a higher-order gravitational field, since graviton loops can couple to the vacuum energy "bubble" graphs which cannot be time-ordered away, i.e. we cannot simply shift the constant vacuum energy,  $E_0$ , such that only  $E' = E - E_0$  is observed.

We shall follow Dirac and Pauli [5, 6] and decompose the real scalar field  $\phi$  according to

$$\phi(x) = \frac{1}{\sqrt{2}} [A(x) + \tilde{A}(x)]. \tag{23}$$

The quantization of A(x) with  $-k_0x_0$  in the phase factor occurs in the usual way, corresponding to positive energy particles, while the other part with  $+k_0x_0$  in the phase factor is quantized such that it leads to negative energy particles. We set

$$A(x) = V^{-1/2} \sum_{k} (2k_0)^{-1/2} \{ A_+(k) \exp[i(\vec{k} \cdot \vec{x} - k_0 x_0)]$$

$$+ A_-(k) \exp[i(-\vec{k} \cdot \vec{x} + k_0 x_0)] \},$$

$$\tilde{A}(x) = V^{-1/2} \sum_{k} (2k_0)^{-1/2} \{ \tilde{A}_+(k) \exp[i(-\vec{k} \cdot \vec{x} + k_0 x_0)]$$

$$(24)$$

$$+\tilde{A}_{-}(k)\exp[i(\vec{k}\cdot\vec{x}-k_{0}x_{0})]\}.$$
 (25)

We now define the Hamiltonian to be

$$H = \int d^3x [\vec{\nabla} \tilde{A} \vec{\nabla} A + \partial_0 \tilde{A} \partial_0 A + \mu^2 \tilde{A} A]. \tag{26}$$

We assume that  $\tilde{A}_{+}(k)$ ,  $A_{+}(k)$  commute with  $\tilde{A}_{-}(k)$ ,  $A_{-}(k)$  and set

$$[A_{+}(k), \tilde{A}_{+}(k)] = 1, \quad [A_{-}(k), \tilde{A}_{-}(k)] = -1,$$
 (27)

leaving the usual commutation relation for  $\tilde{A}(x)$ , A(x) unchanged. We now obtain

$$N_{+}(k) = \tilde{A}_{+}(k)A_{+}(k), \quad N_{-}(k) = -\tilde{A}_{-}(k)A_{-}(k).$$
 (28)

This leads to the energy

$$E = \sum_{k} k_0 \left[ \left( \frac{1}{2} + N_+(k) \right) - \left( \frac{1}{2} + N_-(k) \right) \right] = \sum_{k} k_0 [N_+(k) - N_-(k)]. \tag{29}$$

We see that the zero-point vacuum energy has cancelled in the expression for the total energy E. The underlying symmetry that has cancelled the divergent  $E_0$  is charge conjugation invariance of the vacuum state  $|0\rangle$ . Thus, we have uncovered a symmetry in particle physics that cancels the divergent zero-point vacuum energy. This will still hold in the presence of interactions, in particular, it will hold in the presence of the gravitational coupling of gravitons to the matter fields. Moreover, we can show that for the quantized spin 2 graviton field, the composition of both positive and negative energy gravitons leads to the cancellation of the zero-point graviton vacuum energy and its contribution to  $\Lambda_{\text{vac}}$ .

An alternative quantization procedure consists of defining besides the field  $\phi(x)$  another scalar field  $\chi(x)$ , the adjoint of which is  $\tilde{\chi}(x) = -\chi(x)$ . Then we have

$$\chi(x) = \frac{1}{\sqrt{2}} [A(x) - \tilde{A}(x)],$$
 (30)

with the Fourier decomposition

$$\chi(x) = V^{-1/2} \sum_{k} (2k_0)^{-1/2} \{ \tilde{\chi}(k) \exp[i(\vec{k} \cdot \vec{x} - k_0 x_0)] - \chi(k) \exp[i(-\vec{k} \cdot \vec{x} + k_0 x_0)] \}.$$
 (31)

The  $\chi$  field is quantized according to

$$[\chi(k), \tilde{\chi}(k)] = -1, \tag{32}$$

which gives

$$\tilde{\chi}(k)\chi(k) = -N_{\chi}(k). \tag{33}$$

The Hamiltonian is now given by

$$H = \frac{1}{2} \int d^3x \left[ (\vec{\nabla}\phi)^2 + (\partial_0\phi)^2 + \mu^2\phi^2 - (\vec{\nabla}\chi)^2 - (\partial_0\chi)^2 - \mu^2\chi^2 \right].$$
 (34)

This leads to the energy

$$E = \sum_{k} k_0 [N_{\phi} - N_{\chi}]. \tag{35}$$

As before, the charge conjugation invariance of the vacuum state leads to the cancellation of the zero-point vacuum energy,  $E_0$ . We have

$$\phi(k) = \frac{1}{\sqrt{2}} [A_{+}(k) + \tilde{A}_{-}(k)], \quad \tilde{\phi}(k) = \frac{1}{\sqrt{2}} [\tilde{A}_{+}(k) + A_{-}(k)], \quad (36)$$

$$\chi(k) = \frac{1}{\sqrt{2}} [\tilde{A}_{+}(k) - A_{-}(k)], \quad \tilde{\chi}(k) = \frac{1}{\sqrt{2}} [A_{+}(k) - \tilde{A}_{-}(k)]. \tag{37}$$

The normalization of the state vector  $\Psi$  is given by [6]:

$$\mathcal{N} = \sum_{N_{+}(k), N_{-}(k)} (-1)^{\sum N_{-}(k)} \Psi^{*}(...N_{+}(k)..., ...N_{-}(k)...)$$

$$\times \Psi(...N_{+}(k)...,...N_{-}(k)...) = \text{const.}$$
 (38)

This demonstrates that "negative probability" states will exist with an odd number of particles in states with negative energy. We shall see in the following section, how we can quantize the field in the presence of interactions and avoid a catastrophic instability due to these negative probabilities and negative energy particles. In Section 5, we shall investigate how we can formulate the quantum field theory, so that we obtain a real and positive energy spectrum and a unitary S-matrix.

It can be shown that an equivalent quantization procedure for complex charged scalar fields, neutral vector gauge fields  $A_{\mu}$ , spin-2 graviton fields, and Dirac spinor fields can be derived that leads to the cancellation of the zero-point vacuum energy and the vanishing of  $\Lambda_{\text{vac}}$  in the presence of a gravitational field. This is again due to the charge conjugation invariance of the vacuum state  $|0\rangle$  for these fields.

# 4 Charge Conjugation and Para-Statistics for Negative Energy Bosons in the Vacuum

A spinor field  $\psi$  satisfies in the presence of an electromagnetic interaction the Dirac equation

$$[\gamma^{\mu}(p_{\mu} - eA_{\mu}) - m]\psi = 0, \tag{39}$$

and its charge conjugate equation

$$[\gamma^{\mu}(p_{\mu} + eA_{c\mu}) - m]\psi_c = 0. \tag{40}$$

These equations yield both positive and negative energy solutions of the Dirac equation. The two spinor fields  $\psi$  and  $\psi_c$  and the two photon fields  $A_{\mu}$  and  $A_{c\mu}$  are

associated with positive and negative energy fermions and neutral gauge fields, respectively. For the charge conjugation transformation we have

$$\psi_c = C\gamma^0 \psi^* = C\overline{\psi}^T, \tag{41}$$

where C is the charge conjugation matrix which satisfies

$$C^{-1}\gamma^{\mu}C = -\gamma^{\mu T},\tag{42}$$

and  $\gamma^{\mu T}$  denotes the transpose of the Dirac  $\gamma$  matrix. A similar transformation exists for the gauge field  $A_{\mu}$  which transforms it into the anti-particle gauge field (the photon is its own anti-particle).

Consider the periodic solutions of the positive energy Dirac spinor field  $\psi_{\sigma}^{+}(x)$ :

$$\psi_{\sigma}^{+}(\vec{x}) = \sum_{m} a_{m} \exp(-itE_{m}) u_{m\sigma}^{+}(\vec{x}), \quad \tilde{\psi}_{\sigma}^{+}(\vec{x}) = \sum_{m} \tilde{a}_{m} \exp(itE_{m}) \tilde{u}_{m\sigma}^{+}(\vec{x}), \quad (43)$$

where

$$\int d^3x \tilde{u}_m u_{m'} = \delta_{mm'}. \tag{44}$$

The canonical momentum  $\pi_{\sigma}^{+}(\vec{x})$  is

$$\pi_{\sigma}^{+}(\vec{x}) = i \sum_{m} \tilde{a}_{m} \exp(itE_{m}) \tilde{u}_{m\sigma}^{+}(\vec{x}). \tag{45}$$

Then, the anti-commutator quantization is

$$\{\psi_{\sigma}^{+}(\vec{x},0), \pi_{\sigma'}^{+}(\vec{x'},0)\} = i\delta_{\sigma\sigma'}\delta(\vec{x} - \vec{x'}). \tag{46}$$

We now introduce a negative energy spinor field  $\psi_{\sigma}^{-}(x)$ :

$$\psi_{\sigma}^{-}(\vec{x}) = \sum_{m} b_{m} \exp(itE_{m}) u_{m\sigma}^{-}(\vec{x}), \quad \tilde{\psi}_{\sigma}^{-}(\vec{x}) = \sum_{m} \tilde{b}_{m} \exp(-itE_{m}) \tilde{u}_{m\sigma}^{-}(\vec{x}).$$
 (47)

The canonical momentum is given by

$$\pi_{\sigma}^{-}(\vec{x}) = i \sum_{m} \tilde{b}_{m} \exp(-itE_{m}) \tilde{u}_{m\sigma}^{-}(\vec{x}), \tag{48}$$

and the anti-commutation rule is

$$\{\psi_{\sigma}^{-}(\vec{x},0), \pi_{\sigma'}^{-}(\vec{x'},0)\} = -i\delta_{\sigma\sigma'}\delta(\vec{x} - \vec{x'}). \tag{49}$$

The total energy is given by

$$E = \sum_{m} E_m [N_m^+ - N_m^-]. \tag{50}$$

The zero-point vacuum energy has cancelled for the fermions.

The very existence of negative energy solutions for electrons motivated Dirac to introduce his "hole" theory [21]. This theory prevents a catastrophic instability of the vacuum, which prevents atomic electrons from making radiative transitions into negative energy states. The rate at which electrons make a transition into the negative energy interval -m to -2m is

$$\mathcal{R} \sim \frac{2\alpha^6 m}{\pi} \sim 10^8 \,\text{sec}^{-1}.\tag{51}$$

The rate blows up if all the negative energy levels are taken into account. The stability of the hydrogen ground state is guaranteed by using the Pauli exclusion principle. All the negative energy fermion levels of the vacuum are filled up and all positive energy levels are empty. The stability of the hydrogen-atom ground state is assured, since no more electrons can be accommodated in the negative energy sea by the Pauli exclusion principle. A "hole" in the negative energy sea of fermions is now an anti-particle fermion (for the electron it is a positron). It is possible for a negative energy fermion to absorb radiation and be excited into a positive energy state, when we observe a fermion of charge -|e| and energy +E and in addition a hole in the negative energy sea leading to pair production. If we denote the energy levels of positive and negative energies by E>0 and E<0, respectively, then according to Fermi-Dirac statistical weights the occupation numbers are restricted to the values 0 or 1. Then, the vacuum is described by all negative energy levels being occupied and all positive energy levels being empty:  $N_{fm}=1$  for  $E_{fm}<0$  and  $N_{fm}=0$  for  $E_{fm}>0$ .

In standard second quantized field theory and the interpretation of current particle physics, we obtain a consistent particle quantum field theory by interpreting creation and annihilation of a negative energy particle as the annihilation and creation of a positive energy particle. The negative energy particle vacuum is empty as in the vacuum of a positive energy solution.<sup>1</sup>

We must now also assure the stability of the vacuum against the cascade of positive energy bosons into negative energy states and then into the ground state. We do this by postulating that negative energy bosons in the vacuum satisfy parastatistics. All the negative energy para-boson levels in the vacuum are filled and all the positive energy para-boson levels are empty. The negative energy bosons including the photon and the graviton obey a para-Pauli exclusion principle. We now have for the bosons the energy

$$E = \sum_{k} k_0 \left[ \left( \frac{1}{2} + N_{bk}^+ \right) - \left( \frac{1}{2} + N_{bk}^- \right) \right] = \sum_{k} k_0 [N_{bk}^+ - N_{bk}^-].$$
 (52)

As before, the zero-point vacuum energy has cancelled. However, due to the boson negative energy, para-statistics the boson part of the vacuum is stable.

Let us assume that a field theory satisfies the properties:

<sup>&</sup>lt;sup>1</sup>A negative energy boson vacuum Dirac sea has been investigated by Habara, Nielsen and Ninomiya, and compared with a supersymmetric point of view [22].

- 1. It is invariant under proper Lorentz transformations,
- 2. Its Hamiltonian is Hermitian,  $H^{\dagger} = H$ ,
- 3. The usual connection between spin and statistics is made,
- 4. The field operators are local.

Then it can be shown that such a field theory is invariant under the product operation TCP [23, 24, 25]. We observe that in our field theory formulation our Hamiltonian is not Hermitian but satisfies the adjoint symmetry,  $\tilde{H} = H$ . We may expect that TCP invariance and Lorentz invariance will be broken at some high scale of energy. However, this is an issue that requires further investigation.

The interpretation of the filled negative energy boson sea in the vacuum is based on the para-statistics introduced by Green [26] and studied further by Greenberg and Messiah [27], Drühl, Haag and Roberts [28] and Streater and Wightman [25]. A para-boson field is defined to be of order p:

$$\phi(x) = \sum_{i=1}^{p} \phi^{(i)}(x), \tag{53}$$

where if x and y are spacelike separated  $(x-y)^2 < 0$ , then we have

$$[\phi^{(i)}(x), \phi^{(i)}(y)] = 0, \quad \{\phi^{(i)}(x), \phi^{(j)}(y)\} = 0 \text{ if } i \neq j.$$
(54)

The para-statistics field theory for bosons does not agree with the Lüders-Pauli statistics theorem. When a positive energy boson satisfying normal Bose-Einstein statistics tries to transit into a negative energy state, a para-Pauli exclusion principle prohibits this from happening. It was shown by Greenberg and Messiah [27] that restrictions on the form of the interaction Hamiltonian density,  $H_I$ , are derived by requiring that  $H_I$  be a para-operator. From this restriction on  $H_I$ , selection rules are derived for the S-matrix which prohibit all reactions in which the total number of para particles of order p > 1 in the initial and final states is 1. This selection rule together with experimental input, leads to the conclusion that para-bosons with positive energy will not be observed. This picture of filled negative energy parabosons in the vacuum and the corresponding positive energy para-anti-bosons will assure that the boson part of the vacuum is stable against catastrophic collapse.

The negative energy gravitons will also fill a sea of negative energy levels in the vacuum corresponding to gravitons obeying para-statistics. Selection rules for the graviton-graviton and graviton-matter interactions will prohibit positive energy para-gravitons from being observable.

## 5 Pseudo-Hermitian Hamiltonian and Unitarity

In standard quantum field theory the Hamiltonian is Hermitian,  $H^{\dagger} = H$ , and we are assured that the energy spectrum is real and that the time evolution of the

operator  $U = \exp(itH)$  is unitary and probabilities are positive and preserved for particle transitions. However, in recent years there has been a growth of activity in studying quantum theories with pseudo-Hermitian Hamiltonians, which satisfy the generalized property of adjointness,  $\tilde{H} = \eta^{-1}H^{\dagger}\eta$ , associated with an indefinite metric in Hilbert space [10, 11, 12, 13, 14, 15, 16, 17].

Spectral positivity and unitarity can in special circumstances follow from a symmetry property of the Hamiltonian in terms of the symmetry under the operation of  $\mathcal{PT}$ , where  $\mathcal{P}$  is a linear operator represented by parity reflection, while  $\mathcal{T}$  is an anti-linear operator represented by time reversal. If a Hamiltonian has an unbroken  $\mathcal{PT}$  symmetry, then the energy levels can in special cases be real and the theory can be unitary and free of "ghosts". The operation of  $\mathcal{P}$  leads to  $\vec{x} \to -\vec{x}$ , while the operation of  $\mathcal{T}$  leads to  $i \to -i$  (or  $x^0 \to -x^0$ ). It follows that under the operation of  $\mathcal{PT}$  the Hamiltonian H in (26) is invariant under the  $\mathcal{PT}$  transformation, provided  $\tilde{H} = H$  and  $A_+\tilde{A}_- = A_-\tilde{A}_+$ , which is necessary but not sufficient to assure the reality of the energy eigenvalues.

The proof of unitarity follows from the construction of a linear operator C. This operator is used to define the inner product of state vectors in Hilbert space:

$$\langle \Psi | \Phi \rangle = \Psi^{\mathcal{CPT}} \cdot \Phi. \tag{55}$$

Under general conditions, it can be shown that a necessary and sufficient condition for the existence of the inner product (55) is the reality of the energy spectrum of H [18, 19]. With respect to this inner product, the time evolution of the quantum theory is unitary. In quantum mechanics and in quantum field theory, the operator  $\mathcal{C}$  has the general form

$$C = \exp(Q)\mathcal{P},\tag{56}$$

where Q is a function of the dynamical field theory variables. The form of  $\mathcal{C}$  must be determined by solving for the function Q in terms of chosen field variables and field equations. The form of  $\mathcal{C}$  has been calculated for several simple field theories, e.g.  $\phi^3$  theory and also in massless quantum electrodynamics with a pseudo-Hermitian Hamiltonian. The solution for  $\mathcal{C}$  satisfies

$$C^2 = 1, \quad [C, \mathcal{P}T] = 0, \quad [C, H] = 0. \tag{57}$$

We shall not attempt to determine a specific generalized charge conjugation operator  $\mathcal{C}$  in the present work.

It has also been shown that a special form of  $\mathcal{P}$  leads to a Lorentz invariant scalar expression for the operator  $\mathcal{C}$  [14].

## 6 The Resolution of the Cosmological Constant Problem

We shall postulate that

$$\Lambda_{0\text{eff}} = \Lambda_0 - \overline{\Lambda}_0 = 0. \tag{58}$$

The vanishing of the zero-point vacuum energy in our quantum field theory, including higher-order graviton tree-graph couplings and loops, then assures that (58) is protected against all higher order radiative vacuum corrections.

If we assume that a spontaneous symmetry breaking of the charge conjugation invariance (or  $\mathcal{C}$  invariance of the vacuum) occurs, then this will create a small "observed", effective cosmological constant  $\Lambda_{\rm eff}/8\pi G \sim (2\times 10^{-3}\,eV)^4$ , needed to provide a cosmological constant explanation of the accelerating expansion of the universe [29, 30, 31]. However, it is possible that the accelerating expansion of the universe can be explained by a late-time inhomogeneous cosmological model [32], in which the cosmological constant  $\Lambda_{\rm eff}=0$  and there is no need for a negative pressure "dark energy".

A Casimir vacuum energy has been experimentally observed. In our quantum field theory, the vanishing of the zero-point vacuum energy is only valid in the absence of material boundary conditions as are necessary for the Casimir effect [33]. When material boundary conditions such as the parallel metal plates required to perform the Casimir experiments are imposed, then we can no longer demand that the generalized charge conjugation  $\mathcal{C}$  invariance of the vacuum state is preserved; the breaking of C invariance of the vacuum will produce a non-vanishing zero-point energy effect.

Jaffe [33] points out that the Casimir effect gives no more or less evidence for the "reality" of the vacuum fluctuation energy of quantum fields than any other one-loop effect in quantum electrodynamics, e.g. the vacuum polarization effect associated with charges and currents in atomic physics. Like all other observable effects in quantum electrodynamics, the Casimir effect vanishes as the fine structure constant  $\alpha$  goes to zero.

#### 7 Conclusions

We have formulated a quantum field theory based on an indefinite metric in Hilbert space with a generalization of the Hermitian operator  $H = H^{\dagger}$  to an adjoint operator  $\tilde{H} = \eta^{-1}H^{\dagger}\eta$  and we have  $\tilde{H} = H$ . The quantization of fields in the presence of gravity is performed with a positive and negative energy particle interpretation, which leads to the cancellation of the zero-point vacuum energy due to the generalized charge conjugation  $\mathcal{C}$  invariance of the vacuum:

$$\langle 0|H|0\rangle = 0. (59)$$

We postulate that the effective, classical "bare" cosmological constant  $\Lambda_{0\text{eff}} = 0$ . The condition (59) leads to a protection of the vanishing of the effective cosmological constant,  $\Lambda_{\text{eff}}$ , from all higher order gravitational and external field quantum corrections.

It is assumed that all negative energy levels of fermions are filled in the vacuum, according to Dirac's negative energy sea theory, so that a catastrophic instability of

the fermion part of the vacuum is prevented due to the Pauli exclusion principle. To prevent a similar catastrophic instability of the boson part of the vacuum, we postulate that the negative energy bosons in the vacuum satisfy para-statistics, whereby all the negative para-boson energy levels in the vacuum are filled and all positive energy para-boson levels are empty. A para-Pauli exclusion principle prevents an instability of the boson part of the vacuum. The positive energy normal bosons satisfy the usual Bose-Einstein commutator quantization rule. It is possible to introduce the idea of supersymmetry into our proposed Dirac hole theory of bosons and fermions, for supersymmetric bosons and fermions are treated on the same footing. In the present work we have kept to a non-supersymmetric interpretation.

The indefinite Hilbert space state vector metric can generate negative probabilities for the transitions of particles and violate the unitarity of the S-matrix. To guarantee positive probabilities and the unitarity of transition and scattering amplitudes, we incorporate the  $\mathcal{PT}$  operation on field operators and the action. The generalized charge conjugation operator  $\mathcal{C}$ , introduced by Bender and collaborators [10, 11, 12, 13, 15] is invoked to guarantee that the energy spectrum for gravity and the standard model particle theory is positive, and assure that probabilities are positive and conserved and that the S-matrix is unitary.

The possibility that the new quantum field theory can lead to a perturbatively renormalizable quantum gravity theory will be investigated in a future publication.

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# References

- [1] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
- [2] A. D. Linde, Phys. Lett. **B 200**, 272 (1988).
- [3] A. Linde, Science and Ultimate Reality: From Quantum to Chaos, honoring John Wheeler's 90th birthday, eds. J. D. Barrow, P. C. W. Davies, and C. L. Harper, Cambridge University Press, (2003), hep-th/0211048.
- $[4]\,$  D. E. Kaplan and R. Sundrum, hep-th/0505265.
- [5] P. A. M. Dirac, Proc. Roy. Soc. **180**, 1 (1942).
- [6] W. Pauli, Rev. Mod. Phys. **15**, 175 (1943).
- [7] E. C. G. Sudarshan, Phys. Rev. **123**, 2183 (1961).

- [8] H. J. Schnitzer and E. C. G. Sudarshan, Phys. Rev. 123, 2193 (11961).
- [9] A. M. Gleeson and E. C. D. Sudarshan, Phys. Rev. **D1**, 474 (1970).
- [10] C. M. Bender and S. Boettcher, Phys. Rev. Lett 80, 5242 (1998).
- [11] C. M. Bender, D. C. Brody and H. F. Jones, Phys. Rev. Lett. 89, 270402 (2002).
- [12] C. M. Bender, D. C. Brody and H. F. Jones, Phys. Rev. D70, 025001 (2004), hep-th/0402183.
- [13] C. M. Bender, D. C. Brody, and H. F. Jones, Phys. Rev. Lett. 93, 251601 (2004).
- [14] C. M. Bender S. F. Brandt, Jun-Hua Chen, and Qinghai Wang, Phys. Rev. D71 065010 (2005), hep-th/0412316.
- [15] C. M. Bender, I. Cavero-Pelaez, K. A. Milton, and K. V. Sajesh, Phys. Lett. B613, 97 (2005), hep-th/0501180.
- [16] F. Kleefeld, hep-th/0408028; hep-th/0408097.
- [17] M. Znojil, hep-th/0408081.
- [18] A. Mostafazadeh, J. Math. Phys. 43, 2814 (2002); ibidem 43, 3944 (2002); quant-ph/0508195.
- [19] M. S. Swanson, J. Math. Phys. **45**, 585 (2004).
- [20] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics*, McGraw-Hill Book Co. New York (1964).
- [21] P. A. M. Dirac, Proc. Roy. Soc. 126, 360 (1930); P. A. M. Dirac, Proc. Camb. Phil. Soc. 26, 361 (1930).
- [22] Y. Habara, H. B. Nielsen and M. Ninomiya, hep-th/0504173.
- [23] G. Lüders, Ann. of Phys. (New York), **2**, 1 (1957).
- [24] W. Pauli, Niels Bohr and the Development of Physics, McGraw-Hill, New York, 1955.
- [25] R. F. Streater and A. S. Wightman, *PCT*, *Spin and Statistics*, and *All That*, Princeton University Press, Princeton N. J, 2000.
- [26] H. S. Green, Phys. Rev. **90**, 270 (1953).
- [27] O. W. Greenberg and A. M. I. Messiah, Phys. Rev. B138, 1155 (1965).

- [28] K. Drühl, R. Haag and J. E. Roberts, Commun. Math. Phys. 18, 204 (1970).
- [29] S. Perlmutter et al. Ap. J. 517, 565 (1999); A. G. Riess, et al. Astron. J. 116, 1009 (1998); P. M. Garnavich, et al. Ap. J. 509, 74 (1998).
- [30] A. G. Riess, et al., Ap. J. **607**, 665 (2004), astro-ph/0402512.
- [31] C. L. Bennett et al., Ap. J. Suppl. 148, 1 (2003), astro-ph/0302207; D. N. Spergel et al., Ap. J. Suppl. 148, 175 (2003), astro-ph/0302209.
- [32] J. W. Moffat, astro-ph/0505326.
- [33] R. L. Jaffe, Proc. of Quantum Field Theory Under the Influence of External Conditions, QFEXT03, University of Oklahoma, Norman, Oklahoma, September 15-19, 2003, editor K. A. Miltn, published by Rinton Press, hep-th/0307014; hep-th/0503158.